SPIN-ORBIT VIBRATIONS IN NUCLEI

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A new mechanism of monopole vibrations of nuclei, generated by a two-particle spin-orbit force is suggested. The 0^+ states of this character are estimated to lie below the two-quasiparticle threshold and have strong E0 transitions into the ground state. They can be rather strongly excited in two-particle transfer reactions.

The 0^+ states already found experimentally in spherical as well as deformed nuclei are apparently outside the framework of known theoretical models. This makes the search for new mechanisms of 0^+ vibrations very urgent. We want to point out the possibility of vibrations due to a two-particle spin-orbit force

$$G_{ls} = -\tfrac{1}{2}\kappa(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)\cdot(\boldsymbol{p}_1 - \boldsymbol{p}_2)\times\nabla_1\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)\,, \quad (1)$$

which is known to produce the spin-orbit correction for single-particle levels

$$\Delta\epsilon \approx (\boldsymbol{l}\boldsymbol{\sigma})\frac{1}{r}\frac{\mathrm{d}U(r)}{\mathrm{d}r}\,.$$

Here $U(r)$ is a density-like function. The physical significance of "spin-orbit vibrations" is very clear; it is the surface density (gradient) which is vibrating. This leads (due to the spin-orbit force) to a shift of single-particle levels and a redistribution of nucleons, which in turn changes the surface density.

To simplify the analysis and clarify the physical interpretation it is useful to extract from eq. (1) the part which is basic for our prupose, and approximate it by a separable term.

We restrict ourselves to spherical nuclei and take the average value of eq. (1) for the ground state

$$\sum_{12}\langle 12|G_{ls}^{(+)}|21\rangle\theta_1^2\theta_2^2 = \quad (2)$$

$$= -\kappa\int\mathrm{d}\boldsymbol{r}\sum_1\theta_1^2\langle 1|\boldsymbol{\sigma}\times\boldsymbol{j}(\boldsymbol{r})|1\rangle\cdot\nabla\sum_2\theta_2^2\langle 2|\rho(\boldsymbol{r})|2\rangle.$$

Here $\rho(\boldsymbol{r})$, $\boldsymbol{j}(\boldsymbol{r})$ are the density and current operators. The ground state is supposed to be T-even and so only the part $G_{ls}^{(+)}$, symmetrical with respect to the change $\boldsymbol{p}_2 \to -\boldsymbol{p}_2$, $\boldsymbol{\sigma}_2 \to -\boldsymbol{\sigma}_2$, contributes to eq. (2). The nucleon distribution function θ_ν^2 ($\nu = nljm$) is independent of m and therefore one obtains

$$\sum_{12}\langle 12|G_{ls}^{(+)}|21\rangle\theta_1^2\theta_2^2 = \quad (3)$$

$$= \frac{\kappa}{4\pi}\int r^2\,\mathrm{d}r\sum_1\theta_1^2\langle 1|\boldsymbol{\sigma}\boldsymbol{l}|1\rangle R_1^2(r)\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\sum_2\theta_2^2R_2^2(r)\,,$$

where $\langle\nu|\boldsymbol{\sigma}\boldsymbol{l}|\nu\rangle = j_\nu^2 - l_\nu^2 - \frac{3}{4}$, and $R_\nu(r)$ is the radial part of the single-particle wave function.

We now make the following approximation *

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}R_\nu^2(r) \approx R_\nu^2(R_0)\frac{1}{r}\frac{\mathrm{d}U(r)}{\mathrm{d}r}\,, \quad (4)$$

where $U(r)$ is a universal (state independent) nuclear-potential-like function. In view of eq. (4) we obtain

$$\sum_{12}\langle 12|G_{ls}^{(+)}|21\rangle\theta_1^2\theta_2^2 \approx \quad (5)$$

$$\approx \kappa\sum_1\theta_1^2\langle 1|\boldsymbol{l}\boldsymbol{\sigma}\frac{1}{r}\frac{\mathrm{d}U}{\mathrm{d}r}|1\rangle\sum_2\theta_2^2\langle 2|\frac{1}{4\pi r^2}\delta(r-R_0)|2\rangle.$$

Corresponding to eq. (5) the effective Hamiltonian can be written as

$$H_{ls}^{(+)} = \kappa\sum_1\langle 1|\boldsymbol{\sigma}\boldsymbol{l}\frac{1}{r}\frac{\mathrm{d}U}{\mathrm{d}r}|1\rangle a_1^+a_1\times \quad (6)$$

$$\times\sum_2\langle 2|\frac{1}{4\pi r^2}\delta(r-R_0)|2\rangle a_2^+a_2\,.$$

* The quantity $R_\nu^2(r)$ can be treated as a constant inside the nucleons (fluctuations are averaged in the sum) and as a universally folding function ($\approx U(r)$) on the boundary. Then the dependence of the state comes in only via the amplitude on the surface ($r = R_0$).

Unlike the Q-interaction $H_{ls}^{(+)}$ is represented as a product of two *different* operators. Therefore, for the phenomenological description of the $\boldsymbol{l}\cdot\boldsymbol{s}$ vibrations two collective variables have to be introduced

$$S = \sum_\nu \langle\nu|\boldsymbol{\sigma l}\,\frac{1}{r}\,\frac{dU}{dr}\,|\nu\rangle\theta_\nu^2 \equiv \sum_\nu s_\nu\theta_\nu^2 ,$$
$$D = \sum_\nu \langle\nu|\,\frac{1}{4\pi r^2}\,\delta(r-R_0)\,|\nu\rangle\,\theta_\nu^2 \equiv \sum_\nu d_\nu\theta_\nu^2 . \qquad (7)$$

S determines the energy of the spin-orbit interaction in a standard potential $U(r)$; D describes the surface density. The product $DU(r)$ has then the meaning of a self-consistent potential (at least near the nuclear surface). The interaction (6) contributes to the single-particle energy a self-consistent term

$$\delta\epsilon_\nu = \kappa(s_\nu D + d_\nu S) . \qquad (8)$$

The collective Hamiltonian in S,D variables, $H_{\mathrm{coll}} = T + W(S,D)$ can be obtained by one of the known methods (see e.g. ref. 1). In the harmonic approximation H_{coll} describes two branches of vibrations around the equilibrium values of D and S. The low frequency vibrations turn out, in fact, to be adiabatic and preserve the equilibrium ratio between the variables S and D (defined from $\partial W/\partial S = 0$ or $\partial W/\partial D = 0$). This is what permits the simple interpretation given above.

The microscopic description of vibrations can be given by the homogeneous effective-field equation [2]*

$$\langle 1|\,V^{(+)}\,|1\rangle = \qquad (9)$$
$$= -\sum_2 \langle 12|\,G_{ls}^{(+)}\,|21\rangle\,\frac{2\Delta^2}{E_2(4E_2^2-\omega^2)}\,\langle 2|\,V^{(+)}\,|2\rangle ,$$

where $E_\nu = \sqrt{\epsilon_\nu^2+\Delta^2}$ and ω are the quasiparticle and vibrational energies. In the approximation (5) eq. (9) reduces to the set of two algebraic equations. The solvability condition gives

$$K(\omega^2) \equiv (1+\kappa\langle sd\rangle)^2 - \kappa^2\langle d^2\rangle\langle s^2\rangle \equiv$$
$$\equiv \Big(1 + \kappa\sum_\nu \frac{2\Delta^2 d_\nu s_\nu}{E_\nu(4E_\nu^2-\omega^2)}\Big)^2 + \qquad (10)$$
$$- \kappa^2 \sum_\nu \frac{2\Delta^2 d_\nu^2}{E_\nu(4E_\nu^2-\omega^2)} \sum_\nu \frac{2\Delta^2 s_{\nu'}^2}{E_{\nu'}(4E_{\nu'}^2-\omega^2)} = 0 .$$

* The quantity $G_{ls}^{(+)}$ is the only interaction included in eq. (9), so the effective field $V^{(+)}$ is of particle-hole, T-even type (see eq. (9)). The non-diagonal matrix elements in eq. (9) are omitted. The quantities $s_{\nu\nu'}$, $d_{\nu\nu'}$, $(\nu \neq \nu')$ differ from zero only for transitions $n' = n \pm 2$ which are of no importance.

Here the quantities S and D must be either considered as given parameters (estimated from a comparison of eq. (8) with level energies in a model potential well), or defined self-consistently from eqs. (9) and (8) together with $\theta_\nu^2 = \frac{1}{2}(1-\epsilon_\nu/E_\nu)$. It is easily seen ‡ that eq. (10) always has a solution with the energy below the two-quasiparticle threshold.

For the excited state $|\omega)$ under consideration one can introduce the phonon operator $\mathfrak{A}^+$, defined as $|\omega) = \mathfrak{A}^+|0)$, and expressed through quasiparticle operators α_ν, α_ν^+, as [2]

$$\mathfrak{A}^+ = \sum_\nu \{\mathscr{A}_\nu^{(+)}(\alpha_{\tilde\nu}^+\alpha_\nu^+ - \alpha_\nu\alpha_{\tilde\nu}) + \mathscr{A}_\nu^{(-)}(\alpha_{\tilde\nu}^+\alpha_\nu^+ + \alpha_\nu\alpha_{\tilde\nu})\} ,$$
$$\mathscr{A}_\nu^{(+)} = \frac{E_\nu}{\omega}\,\mathscr{A}_\nu^{(-)} = \qquad (11)$$
$$= \frac{\kappa}{8\pi}\Big(\omega\,\frac{dK}{d\omega^2}\Big)^{-\frac{1}{2}}\,\frac{2\Delta^2}{4E_\nu^2-\omega^2}\,\{\langle s^2\rangle^{\frac{1}{2}} d_\nu - \langle d^2\rangle^{\frac{1}{2}} s_\nu\} .$$

The states ν and $\tilde\nu$ are time-conjugated and all the ω-dependent quantities in eq. (11) are understood as taken for the vibration energy $K(\omega^2) = 0$. As can be estimated from eq. (11) E0 seems to be the main transition from $|\omega)$ into the ground state and two-nucleon transfer reactions are apparently the proper means to excite the $\boldsymbol{l}\boldsymbol{s}$-vibrations.

Here we have considered an simplified model of pure $\boldsymbol{l}\cdot\boldsymbol{s}$-vibration. What should we expect in a more realistic model?

(i) When not using the approximation (4) one has to seek the solution of eq. (9) in the form $V^{(+)} = \alpha(r) + (\boldsymbol{l}\cdot\boldsymbol{\sigma})\beta(r)$ and then to solve numerically the integral equations for the two functions $\alpha(r)$, $\beta(r)$ (or, equivalently, for their matrix elements).

(ii) Additional forces (e.g. central and pairing being included in eq. (9) will lead to the interference of the $\boldsymbol{l}\cdot\boldsymbol{s}$-vibration with other monopole excitations.

All this would complicate the detail of the calculation but the physical picture would not be changed very much. The vibrations of the type considered also exist in deformed nuclei where they can interfere with the β-vibrations. (Here, however, an additional interaction appears since the last sum in eq. (2) depends not only on $|\boldsymbol{r}|$ as in eq. (6) but on angles as well).

‡ Note that the left-hand side $K(\omega^2)$ is positive for $\omega^2 = 0$ which is just the stability condition of the ground state $|0)$.

References

1. S.T. Belyaev, Mat. Fys. Kgl. Medd. Dan. Vid. Selsk. 31, no. 11 (1959).
2. S.T. Belyaev, Yad. Fiz. 4 (1966) 936.