Reprint from

"NUCLEAR STRUCTURE"

INTERNATIONAL ATOMIC ENERGY AGENCY
VIENNA, 1968

THE MICROSCOPIC MODELS OF COLLECTIVE EXCITATIONS
Present status and possible ways of improvement

S.T. BELYAEV
INSTITUTE OF NUCLEAR PHYSICS, NOVOSIBIRSK,
UNION OF SOVIET·SOCIALIST REPUBLICS

Abstract — Аннотация

THE MICROSCOPIC MODELS OF COLLECTIVE EXCITATIONS: PRESENT STATUS AND POSSIBLE WAYS OF IMPROVEMENT. The paper analyses possible generalizations of the known collective excitation models in the following directions: (a) inclusion of other kinds of interactions, generating new types of oscillations, (b) improvement of the quasiboson approximation.

The classification of excitations according to the quantum numbers of correlate pairs provides the basis for the corresponding classification of the elementary interactions generating these excitations. For low excitations the number of important "generating interactions" is not large and practically all of them have been tested.

A special study is made of the particle-particle channel where "elementary interactions" can be introduced without any new parameters, if the hypothesis of gauge invariance of nucleon-nucleon interaction is used. Additional (to usual pairing) interaction in the particle-particle channel generates 0^+-states – "coherent pairing fluctuation" – having a uniquely large probability ratio of E0 and E2 transitions into the ground band. The experimental identification of such states is of great interest.

The necessity to reject the quasiboson approximation is dictated by the complicated vibrational-rotational structure of experimental nuclear spectra. The main difficulty of their theoretical description is the absence of a consistent microscopic picture of rotational excitations. Only in very special models can the group theory be applied and show how rotation appears in the system of interacting particles. A method worked out by Belyaev and Zelevinsky is considered, which gives the possibility of a microscopic description of rotations. The proposed method includes some aspects of group theory approach that, together with the equations of motion for pair operators, allow the consideration of realistic models in a sufficiently good approximation. The illustration of the method serves to obtain the rotational spectrum for j^N configuration with a further generalization on an arbitrary number of levels (in the model with quadrupole interaction). The microscopic structure of rotational states obtained corresponds to the coherent contribution of all the j-levels in rotation and can be directly linked with a visual picture of a deformed nucleus. The wave-function found does not confirm the "stretch scheme" model of Danos and Gillet.

МИКРОСКОПИЧЕСКИЕ МОДЕЛИ КОЛЛЕКТИВНЫХ ВОЗБУЖДЕНИЙ: СОВРЕМЕННОЕ СОСТОЯНИЕ И ВОЗМОЖНОСТИ УСОВЕРШЕНСТВОВАНИЯ. Анализируются возможные обобщения известных моделей коллективных возбуждений в направлении: а) включения других типов взаимодействий, генерирующих новые типы колебаний, и б) обобщения квазибозонного приближения. Классификация возбуждений в соответствии с квантовыми числами коррелированных пар позволяет выделить соответствующие "элементарные взаимодействия", генерирующие эти возбуждения. Для низких возбуждений число этих "генерирующих взаимодействий", невелико, и практически все они рассмотрены. Специально исследуется канал частица-частица, где "элементарные взаимодействия" могут быть введены без каких-либо параметров, если использовать градиентную инвариантность нуклон-нуклонного взаимодействия. Дополнительное (к обычному спариванию) взаимодействие в канале частица-частица генерирует 0^+ состояния ("когерентные флуктуации спаривания"), имеющие уникально большое отношение вероятностей E0- и E2-переходов в основную вращательную полосу. Экспериментальное обнаружение таких состояний представляет большой интерес. Необходимость отказа от квазибозонного приближения диктуется сложной вращательно-колебательной структурой наблюдаемых ядерных спектров. Основная трудность единого теоретического описания таких спектров заключается в отсутствии последовательной микроскопической картины вращательных состояний. Только в очень специальных моделях, при использовании методов теории групп, можно проследить появление вращательных возбуждений в системе взаимо-

действующих частиц. Излагается метод микроскопического описания вращений, разработанный Зелевинским и автором. Метод содержит некоторые аспекты теоретико-группового подхода в сочетании с уравнениями движения для парных операторов и допускает рассмотрение реалистических моделей в достаточно хорошем приближении. Для иллюстрации метода рассматривается возникновение вращательного спектра в модели с квадрупольными силами для конфигурации j^N с дальнейшим обобщением на произвольное число уровней. Полученная микроскопическая структура вращательных состояний соответствует когерентному вкладу всех j-уровней и может быть непосредственно связана с наглядными представлениями о деформированном ядре. Найденная волновая функция не подтверждает модель "stretch scheme" Даноса и Жилле.

In recent years, owing to the progress made in experimental techniques, our knowledge of the spectrum of low-lying excited states of nuclei has significantly increased and become more precise. But the new data obtained make the classification and the quantitative description of even-even nuclei within the framework of traditional models increasingly difficult. We shall dwell on only some of the contradictions.

Classifying the nuclei into deformed and spherical according to their experimental characteristics becomes more and more conditional. "Strange" qualities are found in the traditionally spherical nuclei: large quadrupole moments in 2^+ states, rotational bands in higher excited states.

The multiformity of collective excitations, both in spherical and in deformed nuclei, turns out to be richer than foreseen by the model with five quadrupole collective degrees of freedom. Particularly unclear is the nature of a large number of collective 0^+ states, found in many nuclei in the region of 1-2 MeV.

These and similar anomalies necessitate a critical reappraisal of the arsenal of theoretical models and an analysis of the ways and means of improving and generalizing them.

I

Three groups of collective excitation models can be differentiated. Their main features are summed up in Table I.

While not belittling the euristic value of the first and third model groups, it must be admitted that the microscopic approach is the most consistent and fundamental. Its restrictions are connected with two "weak" points:

(a) Choice of effective interaction;
(b) Quasiboson approximation (or RPA, etc.).

These two circumstances are analysed in the following pages.

TABLE I. GROUPS OF COLLECTIVE EXCITATION MODELS

Phenomenological approach

Main supposition: adiabaticity of collective excitations.

Starting point: postulation of collective Hamiltonian $H_{coll} = T + U$.

Differences between models: collective variables, parameters and form of H_{coll}.

Aim: the description of the greater part of the spectrum by the least number of parameters.

Methods: the solution of the Schrödinger equation with H_{coll}.

Desirable feature: the establishment of general relations between various spectrum characteristics.

Comparison with experiment: a significant part of the spectrum, individual parameters for each nucleus.

Microscopic approach

Starting point: Hamiltonian of nucleons H_p with effective interaction V_{eff}.

Main suppositions: restrictions to (mainly) pair correlations.

Differences between models: choice of single particle functions; form of V_{eff}.

Method: RPA, two-particle Green functions, TDHF, quasiboson approximation, linearized equations of motion, Bogolyubov generalized transformations.

Aims: (a) calculation of phenomenological model parameters;
(b) calculation of H_{coll} for phenomenological models;
(c) definition of the characteristics of the first excited states (collective and 2-quasiparticle).

Comparison with experiment: change of characteristics of the lower levels from one isotope and element to another.

Specific microscopic models

Main suppositions: specially chosen interactions; restricted configuration j^N,$(sd)^N$, etc.).

Aim: precise solution or spectrum classification.

Method: group theory, exact diagonalizations of the Hamiltonian.

Comparison with experiment: only qualitative (for restricted nuclei groups when postulating pure configurations).

II

In the quasiboson approximation the collective excitations of nuclei are described as coherent superpositions of states of the particle-hole (ph) or particle-particle (pp) (hole-hole) type and are characterized: by the baryon quantum number [b = 0 for a pair of type (ph) and b = 2 for (pp) or (hh)], by the angular momentum λ, parity π, isospin τ [1], time-parity T [2] and a radial quantum number ν, numbering excitations of one type. The possibility of realizing one specific type and its characteristics is defined by corresponding components of effective interaction (V_{eff}).

As no fundamental theory exists, we are forced to be content with a phenomenological description (parametrization) V_{eff}, either choosing V_{eff} directly in the form of some potential or calculating V_{eff} in a certain approximation, proceeding from the "realistic forces" between the free nucleons or parametrizing V_{eff} within the framework of Fermi liquid theory. All these methods inevitably contain arbitrariness and very often lead to a complicated form of V_{eff}. Owing to these circumstances, the method representing V_{eff} in the form of a sum of separable "interactions" has become very widespread. Each "interaction" models only a definite group of properties V_{eff} – interaction of a pair of particles only in a definite quantum state. The pairing and quadrupole interaction model has proved very fruitful, explaining many regularities in the nucleus structure.

It is convenient to classify "elementary interactions" according to quantum numbers of corresponding pair states. Then the characteristic b = 2 [channels (pp), (hh)] $\lambda = 0$ can be assigned to pairing and b = 0, $\lambda = 2$ to quadrupole interaction. Moreover, the radial quantum number $\nu = 0$, defined as the number of nodes of the radial wave function R(r) of the pair, can be assigned to both interactions R(r) (to pairing $R \sim 1$, for quadrupole interaction $R \sim r^2$). Interaction in the channel particle-hole is also characterized by time parity. For quadrupole forces T = +1.

Within the framework of the pairing + quadrupole interaction model only a restricted class of collective excitations is described: quadrupole (β, γ) oscillations ($\nu b \lambda T = 002^+$) and "pair vibrations" (020^+) [3]. A more general model is needed to describe excitations of a different nature.

For the particle-hole channel (b = 0) we can first of all study higher angular momenta (e.g. actual interaction with $\lambda = 3$ [4]). If we restrict ourselves to quadrupole excitations only ($\lambda = 2$), then only T-odd forces (spin-quadrupole [5]) may be included. Studying the terms with $\nu > 0$ is of little interest, as it leads to too high excitations. It is worthy of note that the ability to generate 0^+ excitation in the particle-hole channel is very restricted as monopole forces ($\lambda = 0$) result only in high excitations.

In the channel particle-particle (b = 2) each type of interaction can generate two branches with different T-parity. Pairing ($\lambda = 0$; $\nu = 0$) in addition to "pair vibrations" can also generate the T-odd branch but the latter has zero energy and hence is a "ghost" excitation. It is only natural to study elementary interactions of the same nature ($\lambda = 0$) but with $\nu > 0$. As for interaction with $\lambda > 0$, for this the main factor is terms with $\nu = 0$. The author has studied the possibilities and the consequences of supplementary "elementary interactions" in the particle-particle channel [2]. The latter, unlike interaction in the (ph)-channel, can be included without introducing additional free parameters if the gauge invariance of the nucleon-nucleon interaction is used.

By the gauge invariance of interaction we mean its non-changeability when transforming the moments of interacting particles $\vec{p}_{1,2} \to \vec{p}_{1,2} - \nabla\Lambda(\vec{x}_{1,2})$, where $\Lambda(\vec{x})$ is an arbitrary co-ordinate function. Potentials not depending on velocity, as well as spin orbital forces of the form

$$U_{ls} \sim (\vec{\sigma}_1 + \vec{\sigma}_2)\cdot(\vec{p}_1 - \vec{p}_2) \times \nabla\, \delta(\vec{x}_1 - \vec{x}_2)$$

have strict invariance. That is why the forces acting between free nucleons may, with good precision, be considered to be gauge-invariant. Though

the transition from the interaction of free nucleons V_0 to the effective interaction of nucleons inside the nuclei V_{eff} is a very complicated problem, we can still formulate the conditions on V_{eff} in the general form for gauge-invariant V_0. The usual pairing does not satisfy these conditions and must be supplemented by a number of other "elementary interactions" whose form is defined in a unique manner [2].

TABLE II. ENERGIES ω, PROBABILITIES OF E2-TRANSITIONS INTO THE GROUND ROTATIONAL BAND (IN SINGLE-PARTICLE UNITS), AND $\rho^2 = |(0|r^2|\omega)|^2/R^4$ FOR THE FIRST TWO 0^+ STATES

The quadrupole interaction constant $\kappa = k\hbar\mathring{\omega}A^{-4/3}$, $k = 18.02$; $e_p = 1 + e_{eff}$, $e_n = e_{eff} = 1$.
(Parameters are taken from the nucleus ^{150}Nd: $\omega = 0.69$; $B(E2) = 5 \pm 1.3$).

	Theory			Experiment	
	ω (MeV)	$B(E2)/B_{sp}$	$\rho^2(E0)$	ω (MeV)	$B(E2)/B_{sp}$
^{152}Sm	0.81	3.6	0.14	0.685	2.5 ± 0.6
	1.69	3×10^{-4}	0.53	1.10	-
^{154}Sm	1.11	2.2	0.19	1.1	1.2 ± 0.3
	1.7	10^{-3}	0.48	1.22	-
^{154}Gd	0.95	2.7	0.08	0.68	4.8 ± 1.2
	1.68	10^{-3}	0.7	-	-
^{156}Gd	1.22	1.8	0.12	1.04	2.8 ± 1.2
	1.68	2×10^{-5}	0.67	-	-
^{156}Dy	1.05	2.2	0.04	0.68	-
	1.71	10^{-5}	0.85	-	-
^{158}Dy	1.29	1.43	0.07	0.99	> 0.3
	1.71	8×10^{-5}	0.84	-	-
^{164}Er	1.49	0.17	0.88	1.245	> 0.05
	1.5	0.16	0.17	1.698	-
^{166}Er	1.5	10^{-2}	0.96	1.46	-
	1.57	0.02	0.14	-	-
^{168}Yb	1.46	0.11	0.83	1.156	-
	1.57	0.01	0.25	1.196	-
^{170}Yb	1.25	10^{-4}	0.22	1.065	-
	1.46	0.11	0.92	-	-
^{174}Hf	1.22	0.08	0.24	0.85	-
	1.55	6×10^{-3}	8×10^{-3}	-	-
^{176}Hf	1.46	5×10^{-3}	0.14	1.25	-
	1.67	0.05	0.06	1.42	-

III

Rumiantsev and the author [6] calculated the collective 0^+ states for deformed nuclei with quadrupole interaction and gauge-invariant pairing (terms with $\lambda\nu$ = 00; 01; 20 were considered[1]). The role of supplementary terms in V_{eff} is as follows:

(a) New branches of collective 0^+ excitations appear, i.e. "coherent pairing fluctuations" [2].

(b) Because of the coupling with new excitations, β oscillations change their properties somewhat. The change of the latter can be represented by the dependence of the effective constant of the quadrupole interaction on energy. Taking this effect into account noticeably improves the agreement with experiment.

Some results of the calculations for the first two 0^+ states are given in Table II.

Especially noticeable is the striking difference of the B(E2) transition probabilities into the ground rotational band (except ^{164}Er). States having a larger B(E2) are slightly modified β oscillations. As a rule these states lie somewhat lower in energy (^{166}Er being an exception). Second states (mainly coherent pairing fluctuations) deserve special attention because of the uniquely large ratio[2] $X = \rho^2/B(E2)$. Finding these states experimentally is most interesting.

It is evident from the classification of pair states that coherent pairing fluctuations exhaust the new types of low-lying excited collective states in the framework of quasiboson approximations. Using a more exact form of V_{eff} can, of course, alter the quantitative characteristics, but it is doubtful whether it will change the qualitative scheme of excitations.

IV

Quasiboson approximation (and its equivalent RPA, etc.) gives one the possibility of considering only two quasiparticle states and their coherent superposition. Thus the results are applicable only to the first (single phonon) excitation. The highest excited states can be constructed out of several non-interacting or weakly interacting phonons. But an approximation of this kind turns out to be rather rough.

One means of improving the model is the simultaneous and equal consideration of two- and four-particle correlations [7]. This gives us the chance of improving the description of the two-phonon states, but an improvement of this sort is hardly worth the complication of the calculations. Further steps in this direction, namely the inclusion of higher states, can hardly be considered promising. A different approach, namely a fundamental change of the quasiboson approximation, seems to be much more promising.

---

[1] Especially important is the fact that supplementary "elementary interactions" are introduced with no new parameters. Their constants are uniquely defined by the demands of gauge invariance.

[2] It is worth mentioning that the too small values of B(E2) in Table II should be considered only as qualifiers as in this case the small corrections that are not considered may essentially change the quantitative result.

In the quasiboson approximation the spectrum of collective excitations is constructed of non-interacting phonons, in other words it is equidistant. It is evident that rotational states drop out of this description. That is why we are forced to describe β, γ oscillations and rotations of deformed nuclei under a microscopic approach separately and by different methods. On the other hand, the analysis of experimental data proves that there does not exist any considerable difference between the rotational and vibrational states. A sequence of nuclei can be chosen in which vibrational states gradually turn into rotational ones [8]. A similar transition can be seen, perhaps, in "spherical" nuclei along with the growth of the energy of excitation [9]. These circumstances force us to look for a unified description of both vibrational and rotational states.

Marumori et al. [10] tried to make the boson rules of commutation for pair quadrupole operators $[A^{(+)}_{2\mu}; A^{(-)}_{2\mu'}] = 2(-)^{\mu}\delta_{\mu,-\mu'}$ more complicated by considering the fact that the commutators of different components $A^{(+)}_{2\mu}$ (or $A^{(-)}_{2\mu}$) are proportional to the angular momentum operator $\hat{J}_{\lambda}$. Finally, the properties of the phonons (of the operator that raises the system one state higher) begin to depend upon the angular momenta of the initial (J) and final (J') states; this leads to the deviation of the spectrum from equidistance. The phonon energy, which is related to the energy of a two-quasiparticle transition (in an isolated j - level model), is equal to

$$\omega_{J'J} = \frac{1}{2}(1-\omega_0^2)\beta_{J'J} + \sqrt{\omega_0^2 + \frac{1}{4}(1-\omega_0^2)^2\beta^2_{J'J}} \qquad (1)$$

where $\beta_{J'J} = \text{const} \cdot [J'(J'+1) - J(J+1)]$ and ω_0 is the phonon energy in a quasiboson approximation, defined by the relation of the dimensionless quadrupole (κ) and pairing (g) interaction constants and by the occupation of the level $n = N/(2j+1)$:

$$\omega_0^2 = 1 - \frac{\varkappa}{g} 4n(1-n) \qquad (2)$$

When a change ω_0^2 in the interval from 1 to 0 takes place, the spectrum defined by Eq.(1) changes from equidistant to purely rotational. But it would be erroneous to think that we have obtained a unique description of both spherical and deformed nuclei as Eq.(1) has no sense in the extreme field of nuclei which are known to be deformed ($\kappa/g \to \infty$; $\omega_0^2 \to -\infty$).

V

The rotational bands of deformed nuclei present the main problem in unifying the microscopic description of nuclear spectra. As is known, when considering only quadrupole forces between nucleons in an oscillatory potential the rotational spectrum can be easily obtained from group properties [11]. But group theory methods turn out to be too complicated and have little prospect for real models of medium and heavy nuclei.

Zelevinsky and the author have studied the possibilities of deriving a rotational spectrum from particle dynamics by other methods; even if they are approximate, they allow a solution in realistic models.

Let us first study the nucleons on one j-level ($j \gg 1$) interacting through quadrupole forces

$$H(j) = -\frac{1}{2}\varkappa \sum_{\mu} (-)^{\mu} Q_{2-\mu}(j)\, Q_{2\mu}(j) \tag{3}$$

If we consider the angular momentum (neglecting the octupole) in the commutator of quadrupole operators

$$\left[Q_{2\mu}(j),\, Q_{2\mu'}(j)\right] = -\sqrt{30}\, b(j) \sum_{\lambda} (-)^{\lambda} \begin{pmatrix} 2 & 2 & 1 \\ \mu & \mu' & -\lambda \end{pmatrix} \hat{J}_{\lambda} \tag{4}$$

then the eight operators $Q_{2\mu}$ and $\hat{J}_{\lambda}$ included in Eqs (3) and (4) generate the group SU(3) and thus the problem becomes absolutely equivalent to the Elliott problem [11]. The system spectrum consists of a set of rotational bands (limited from above) that are characterized by the different values of the Casimir operator

$$C_j = \sum_{\mu} (-)^{\mu} Q_{2-\mu}(j)\, Q_{2\mu}(j) + b(j) \sum_{\lambda} (-)^{\lambda} \hat{J}_{-\lambda} \hat{J}_{\lambda} \tag{5}$$

The lower band has the following value of this quantity:

$$C_j^{0} = \frac{4}{3} b(j) \lambda_j (\lambda_j + 3) \tag{6}$$

where λ_j, the maximum possible value of the system angular momentum, is defined by the level occupation $n_j = N_j/(2j+1)$

$$\lambda_j = \frac{1}{2}(2j+1)^2 n_j (1-n_j) \tag{7}$$

The energies of the levels and the reduced matrix elements of Q in the ground band have the following form:

$$E(J) = E(0) + \frac{1}{2}\varkappa b(j)\, J(J+1) \tag{8}$$

$$(J \| Q(j) \| J') = \sqrt{C_j^{0}(2J+1)(2J'+1)} \begin{pmatrix} J & 2 & J' \\ 0 & 0 & 0 \end{pmatrix} \xi_{JJ'} \tag{9}$$

$$\xi_{JJ}^2 = 1 + \frac{9}{4\lambda_j(\lambda_j+3)}, \qquad \xi_{JJ+2}^2 = 1 - \frac{J(J+3)}{\lambda_j(\lambda_j+3)}$$

The factor ξ, when the angular momenta J are close to the maximum value, cuts non-diagonal matrix elements. If we restrict ourselves to the region $J \ll \lambda_j$, then the deviation of ξ from unity can be neglected[3]. In this case the matrix elements will satisfy Eq.(4) without the right-hand side. An approximation of this kind is fully consistent as the right-hand side of (4) is actually small ($b(j) \sim j^{-3}$).

VI

Let us now formulate a simple approximation method for solving the same problem which would be valid in the region $J \ll \lambda_j$.

(A) Let us neglect the right-hand side of (4) and consider for Q only matrix elements inside the band. We then obtain the whole of Eq.(9) (with $\xi = 1$) but only within the accuracy of the normalization constant.

(B) From Eq.(5), neglecting the last term, let us normalize Q through C_j^0.

(C) The value C_j^0 is obtained by calculating the expectation value of the Casimir operator in the state $J = \lambda_j$ when the last term in Eq.(5) has to be retained, while the non-diagonal matrix element of Q can be neglected.[4]

(D) In order to define the rotational level energies, let us make use of the equations of motion for Q:

$$(E_{J'} - E_J)(J\|Q(j)\|J') = \tfrac{1}{2}\varkappa\sqrt{30}\,\beta(j)(J\|(\hat{J}Q + Q\hat{J})_2\|J') \qquad (10)$$

from which Eq.(8) immediately follows.

VII

For the transition to realistic models the formulated method of approximation demands generalizations in two directions: one should not be restricted to one j-level and one should include other types of interaction. Let us start out from two postulates, physically evident for deformed nuclei:

(a) There exists a sequence of states with angular momenta $J = 0, 2, 4\ldots$ ("rotational band").

(b) The full quadrupole moment operator $Q_{2\mu}$ has large matrix elements inside the band (diagonal and non-diagonal), while the transitions into states of other types are relatively small.

As an illustration let us study an arbitrary number of j-levels with restriction to $Q \cdot Q$ interaction of the following kind

$$H_{QQ} = -\tfrac{1}{2}\varkappa \sum_{jj'\mu} (-)^{\mu} Q_{2-\mu}(j) Q_{2\mu}(j') \qquad (11)$$

---

[3] When $\xi = 1$ the relation between the matrix elements (9) is the same as in the case of the symmetrical rotator.

[4] Thus, having an explicit expression for the Casimir operator, we can relate the regions of small and extremely large angular momenta J.

Then the operations (A), (B), (C) given above lead to formulas (6), (7) and (9) for partial values (defined on each j-level). Equation (10) also remains valid provided that by Q on the right-hand side we understand a full quadrupole moment and by $\hat{J}_\lambda \equiv \hat{J}_\lambda(j)$ a partial angular momentum operator. If we assume $\hat{J}_\lambda(j)$ to be proportional to the operator of the total angular momentum of the system $\hat{J}_\lambda(j) = g(j)\,\hat{J}^{tot}_\lambda$ (inside the rotational band), then both the energy spectrum and the "Lande factor" g(j) are defined from (10):

$$E(J) = E(0) + \tfrac{1}{2}\varkappa \sum_j g(j)\beta(j)\,J(J+1),$$

$$g(j) = \frac{\lambda_j}{\sqrt{\beta(j)}}\left[\sum_{j'}\frac{\lambda_{j'}}{\sqrt{\beta(j')}}\right]^{-1} \qquad (12)$$

The result has a simple physical meaning. The partial momenta of j-levels sum coherently into a full angular momentum, the contribution of each level depending upon the occupation ($\lambda_j \sim n_j(1-n_j)$. The particle distribution among j-levels (n_j) can be defined by minimizing the energy of the ground state:

$$E(0) = \sum_j (2j+1)n_j\varepsilon_j - \tfrac{2}{3}\varkappa\left(\sum_j \lambda_j\sqrt{\beta(j)}\right)^2 \qquad (13)$$

The structure of the ground state can be clearly presented for the case of two j-levels. If their interaction ($Q(j_1)\cdot Q(j_2)$) is fully neglected then the lowest state (when the distribution of particles in levels is fixed) will be $\left|(j_1^{N_1})_{J_1=0}\ (j_2^{N_2})_{J_2=0}\right\rangle$. The higher angular momenta $J_1, J_2 > 0$ are not profitable because of the loss in the rotational energy. But the quadrupole interaction between levels covers this small effect, which is why coherent superposition turns out to be the most profitable combination[5]:

$$|J=0\rangle \sim \sum_{J'} \left|\left((j_1^{N_1})_{J'}\,(j_2^{N_2})_{J'}\right)_0\right\rangle \qquad (14)$$

A similar result could be obtained by selecting wave-functions for each of the j-levels as a "deformed state", $|\vec{n}\rangle = \sum_{J'M'} Y_{J'M'}(\vec{n})\,|J'M'\rangle$, and projecting their product into the state with a definite total angular momentum. The microscopic states thus found actually correspond to the phenomenological suppositions about the deformed nuclei.

The authors have also studied a more realistic model with quadrupole interaction and pairing. In this case one has to take into consideration the operators with higher multipolarity ($Q_{k\mu}$, $k > 2$), as well as pair creation and annihilation operators ($A^\dagger_{k\mu}$, $A_{k\mu}$). In spite of the increase in the number of operators the problem can be treated in a similar manner, provided that

[5] It is worth noting that the given coupling scheme is in principle different from the "stretch scheme" [12] where instead of the coherent sum (14) there ought to be only the one term with a maximal angular momentum.

Eq.(5) is substituted by a more general Casimir operator. The dependence of the matrix elements on angular momenta J, J' inside the rotational band can be separated in the form similar to Eq.(9). Then one obtains, for the amplitudes of the matrix elements, equations of motion analogous to those in the deformed-nucleus model. The moment of inertia found exhibits reasonable dependence on pairing interaction strength. Details of the calculation will be published elsewhere. Results obtained allow one to hope for the successful solution of the last step of the problem: the unified description of both rotational and vibrational excitations.

Let me finish with a quite general remark. Irrespective of the success of the concrete approach described here, the author is positive that for the transition from the description of one elementary excitation (phonon) to the whole spectrum of nucleus excitation, it is necessary to introduce group methods into the traditional microscopic models. Coming back to the collective model classification, this idea can be formulated as follows: combinations of methods, approaches and aims typical for various classes of models are in fact the most promising.

REFERENCES

[1] BOHR, A., Congrès Intern. Phys. Nucl., Vol.1, Paris (1964).
[2] BELYAEV, S.T., Yadernaya Fiz. 4 (1966) 936.
[3] BES, D.R., BROGLIA, R.A., Nucl. Phys. 80 (1966) 289.
[4] SOLOVIEV, V.G., VOGEL, P., KORNEICHUK, A.A., Izvestia Akad. Nauk SSSR 28 (1964) 1599.
[5] CHERNEY, M.I., PYATOV, N.I., ZHELEZNOVA, K.M., Izvestia Akad. Nauk SSSR 31 (1967) 550.
[6] BELYAEV, S.T., RUMIANTSEV, B.A., Proc. Int. Conf. Tokyo (1967).
[7] OTTAVIANI, P.L., SAVOIA, M., SAWICKI, J., TOMASINI, A., Phys. Rev. 153 (1967) 1138.
[8] SHELINE, R.K., Rev. Mod. Phys. 32 (1960) 1.
SAKAI, M., Nucl. Phys. A104 (1967) 301.
[9] PEKER, L.K., Lectures at the 3rd winter school, Leningrad (1968).
[10] MARUMORI, T., SHONO, Y., YAMAMURA, M., TOKUNAGA, A., MIYANISHI, Y., Phys. Lett. 25B (1967) 249.
[11] ELLIOTT, J.P., Proc. Roy. Soc. 245 (1958) 128, 562.
[12] DANOS, M., GILLET, V., Phys. Rev. 161 (1967) 1034.

DISCUSSION

N.I. PYATOV: The numerical results which you have given for 0^+ states can be obtained from the usual model by including spin-quadrupole interactions. Spin-quadrupole interactions put the second 0^+ states below the energy gap, but small B(E2) and large values of X are characteristic of pairing vibrations. Can you definitely point to a feature by which one can distinguish coherent pairing fluctuations from other types of vibrations?

S.T. BELYAEV: I would prefer to put the question in a different way. In order to include spin-quadrupole forces, a new constant must be introduced. The additional interaction, which we considered, follows automatically from the principle of gauge invariance and we must take it into account. As regards spin-quadrupole forces, if any, they should be taken into consideration additionally.

S. SZPIKOWSKI: We know from the Elliott model that the SU(3) group is a proper group to treat the quadrupole-quadrupole forces. On the other hand, the pairing forces are governed by the orthogonal group in five dimensions (j-j coupling) or in eight dimensions (L-S coupling). From the group theory point of view one can do a simple thing in principle: take the operators of Q-Q forces that generate the SU(3) group, and the pairing operators that generate the O_5 (or O_8) group, and commute them to complete the set of generators which will generate the group to be found. Such a group will be a symmetry group of the interactions and would be just a group to diagonalize at the same time these interactions. Do you know anything about the realization of such a program?

S.T. BELYAEV: Exact diagonalization of a similar type is possible only for the simplest single-particle configuration. The generalization to a real scheme of single-particle levels and real forces is practically impossible. The main purpose of our approach is to develop an approximate method for the Elliott model which, however, will permit generalization. This is not a group theory method, although we have borrowed the Kasimir operator from the latter.

I.N. MIKHAILOV: Let me raise a question about the theoretical foundation of the principle of gauge invariance relative to effective forces. Could you give the reason for such an application?

S.T. BELYAEV: The assumption about the gauge invariance is made for the forces between free nucleons and not for the effective interaction of nucleons in a nucleus. But it follows as a consequence that there are certain limitations on the effective interaction.

V.G. SOLOVIEV: Important progress has been made in the many-body problem due to the development of methods in which some law or other is followed approximately. The mathematical basis for applying such methods was created by N.N. Bogolyubov in his paper on quasi-averages. As regards the physical problem, the task is to find such a mathematical method that would permit the most precise solution of the problem. In so doing, one has to restrict oneself by applying one or another conservation law only approximately or for some average values. Certainly, it is good if some conservation law or other is obeyed sufficiently accurately, i.e. gauge invariance takes place. However, the requirement of the fulfilment of gauge invariance in the many-body problem is not strict.

S.T. BELYAEV: I do not agree with the latter statement. Certainly, we are sometimes forced to violate rigorous conservation laws at a certain stage of solving the problem. We do this with the gauge principle, when solving a single-particle problem. (Another example is single-particle levels in the deformed well - the angular momentum is not conserved). However, the conservation laws and symmetry principles must be recovered at further stages of the calculation. Briefly, we must compensate for what we have violated in the single-particle picture (gauge invariance) by collective modes of excitations. These are just the excitations considered.

I.S. SHAPIRO: I would like to ask a simple question: what do you mean in this case by T-parity in the particle-particle system? I can imagine T-parity in the particle-hole system. But as regards the wave-function of two particles, it cannot pass into itself with time.

S.T. BELYAEV: The T-parity of excitation is determined by the relative phase of the contribution of two-particle and two-hole states to

the phonon wave-function, in other words, by the behaviour of the phonon wave-function with respect to the interchange of holes and particles.

I.N. MIKHAILOV: What is the reason for using realistic forces in the theories considered above (say, in the random phase approximation)?

S.T. BELYAEV: The conception of "realistic forces" is usually related to the interaction of free nucleons, whereas collective modes of excitation are determined by the effective interaction inside the nucleus. We used just the latter (certainly, in the form of a model).

B.L. BIRBRAIR: Is there any connection between the pairing coherent fluctuations and the nuclear analogue of ordinary sound? To obtain the latter it is sufficient to take into account the equation of continuity, which in itself is a condition for frequencies, but which does not explicitly include the interaction. Thus, for the sound as well as for the pairing coherent fluctuation there is no need to introduce new constants and therefore there should be a connection between these types of excitation.

S.T. BELYAEV: There actually exists some formal analogy between the sound waves in the superconductor and the excitations considered, although the difference in the systems is very important, which affects even formal results - the frequency of 0^+ vibrations depends on the pairing constant.

D.F. ZARETSKY: Are there vibrations of your type in spherical nuclei? If they do exist, how do they manifest themselves?

S.T. BELYAEV: Similar vibrations should exist in spherical nuclei. We did not calculate them because, in our opinion, the structure of spherical nuclei is not clear at present.

O. NATHAN: I would like to point out that the two second excited states of the Sm isotopes shown in your Table II have a very specific behaviour in double stripping and double pick-up. I would therefore suggest that you use your theory to calculate these (t, p) and (p, t) transition rates. This might test your theory more sensitively than the B(E2) values which are probably very difficult to measure precisely.

S.T. BELYAEV: I agree. At present we are doing such calculations although the results will probably only be qualitative due to the uncertainty of the operator generating (p, t) and (t, p) reactions.

I.N. MIKHAILOV: Is the use of operators of the interaction with the scattering amplitude dependent on density in the Migdal theory consistent with your principle?

S.T. BELYAEV: If the effective interaction depends only on space variables there are no restrictions.

A.S. DAVYDOV: The rotation considered in the Elliott model does not correspond to the nucleus rotation as we usually understand it - the rotation of the coordinate system associated with the directions of the main inertial moments. This is probably the quasiparticle rotation in relation to the coordinate system connected with a self-consistent field, i.e. internal rotation. I shall clarify my remark by an example from molecular theory. The motion of electrons in the field of nuclei of a molecule with a certain total momentum corresponds to their rotation around the molecule core. On the other hand, we can speak of the rotation of the molecule as a whole, i.e. the rotation of the nuclei in the molecule (of the field, in which electrons move).

S.T. BELYAEV: In the Elliott model internal motion and rotation as a whole are not separated. The problem is solved precisely and

self-consistently because both "deformation" (the average value of the total quadrupole moment) and the probabilities of E2-transitions and the energetic spectrum appear simultaneously.

N.I. PYATOV: In your classification of effective interactions in the particle-hole channel there is no interaction with $\lambda = 1$, say $(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$-interactions which may lead to the core spin polarization and generate collective 1^+ states. Have you any particular arguments against the consideration of such effective forces?

S.T. BELYAEV: I have presented only the scheme of the classification, but have not, naturally, pointed out all the possibilities. The terms with $\lambda = 1$ also exist, in principle.

A. BOHR: Would not the gauge invariance put restrictions on effective two-particle forces that could be tested for configurations with two particles or holes outside closed shells?

S.T. BELYAEV: This is probably difficult because the gauge invariance gives rather weak restrictions. Thus arbitrary potentials $V(\vec{r})$ may be used.

A.S. DAVYDOV: Quasiparticles are determined by a self-consistent field. Vibrational modes are connected with a change of the self-consistent field. The consideration of the interaction of quasiparticles with the phonons of surface oscillations should reflect the effect of the change of the self-consistent field.